THERMODYNAMICS OF THE NAMBU – JONA – LASINIO LIKE MODELS AND SCALAR σ MESON

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Introduction

Models of the Nambu and Jona-Lasinio (NJL) type[1-3] have a long story and have been used to describe the dynamics and thermodynamics of the light mesons. Such models give a simple and practical example of the basic mechanism of the spontaneous breaking of chiral symmetry, a key features of QCD at finite temperature and chemical potential. The behavior of a QCD system is governed by symmetry properties of the Lagrangian, namely the global symmetry $SU_L(N_f)\times SU_R(N_f)$ which is spontaneously broken to $SU_V(N_f)$ and the exact $SU_c(N_c)$ local color symmetry. On the other hand, in a non-abelian pure gauge theory, the Polyakov loop serves as an order parameter for the transition from the low temperature confined phase (Z_{N_c} symmetric) to the high temperature deconfined phase characterized by the spontaneous breaking Z_{N_c} symmetry (PNJL model). In the PNJL model quarks are coupled simultaneously to the chiral condensate and to Polyakov loop and the model includes features of both chiral and Z_{N_c} symmetry breaking. The model successfully reproduces lattice data concerning QCD thermodynamics. It is therefore natural to investigate the predictions of the PNJL model for meson properties and their decays[1].

The model

We employ the model with the following Lagrangian

$$L_{PNJL} = \bar{q}(i\gamma_\mu D_\mu - \hat{m}_0)q - G_1[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] - U(\Phi[A],\bar{\Phi}[A];T)$$

where the covariant derivative $D_\mu = \partial_\mu - iA_\mu$ and $A^\mu = \delta_0^\mu A^0$ with $A^0 = -iA_4$. The strong coupling constant is absorbed in the definition of A_μ. At zero temperature the Polyakov loop field Φ and the quark field are decoupled. The quark field $\bar{q} = (\bar{u},\bar{d})$, $\hat{m} = diag(m_u, m_d)$, $\vec{\tau}$ - Pauli matrices acting in the two flavor space. The simple sets of the Polyakov loop field Φ equal its expectation value which minimizes the potential[1]

$$\frac{U(\Phi,\bar{\Phi};T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3(T)}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

where $b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$ and

a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

The grand canonical potential corresponding the PNJL Lagrangian has the form

$$\Omega = U(\Phi,\bar{\Phi};T) + \frac{(m-m_0)^2}{2G_1} - 12N_f\int_\Lambda \frac{d^3p}{(2\pi)^3}E_p - 4T\int_\Lambda \frac{d^3p}{(2\pi)^3}(V_1 + V_2)$$

$$V_1 = \ln\left[1 + 3\left(\Phi + \bar{\Phi}e^{-\beta(E_p-\mu)}\right)e^{-\beta(E_p-\mu)} + e^{-3\beta(E_p-\mu)}\right]$$

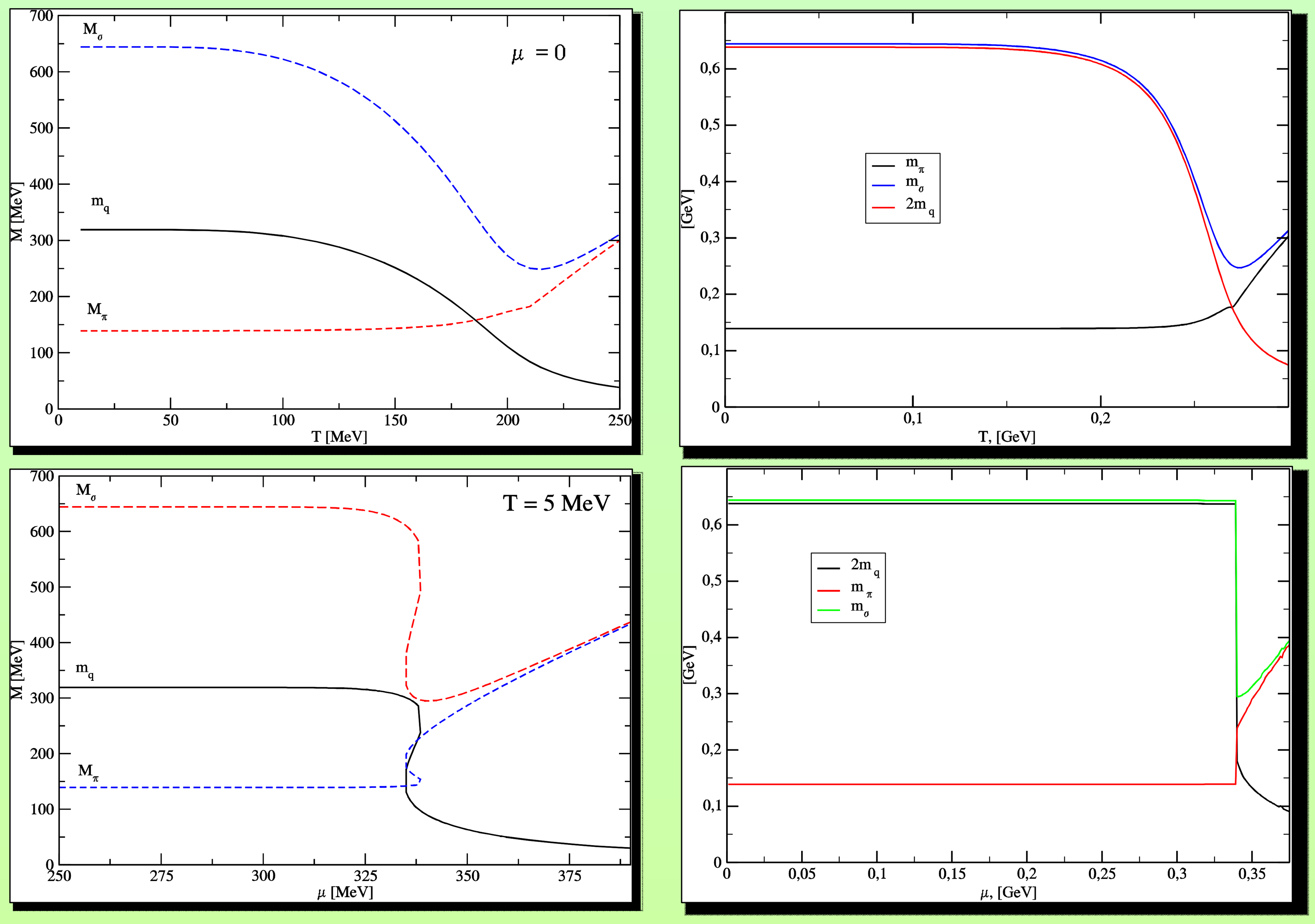
$$V_2 = \ln\left[1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_p+\mu)}\right)e^{-\beta(E_p+\mu)} + e^{-3\beta(E_p+\mu)}\right]$$

Model parameters

Λ [GeV]	G_1 [GeV^{-2}]	m_0 [GeV]	F_π [GeV]	m_π [GeV]
0.639	5.227	0.0055	0.092	0.139

The temperature and chemical potential dependences of the calculated masses and decay widths are presented in figures for NJL and PNJL models.

NJL PNJL

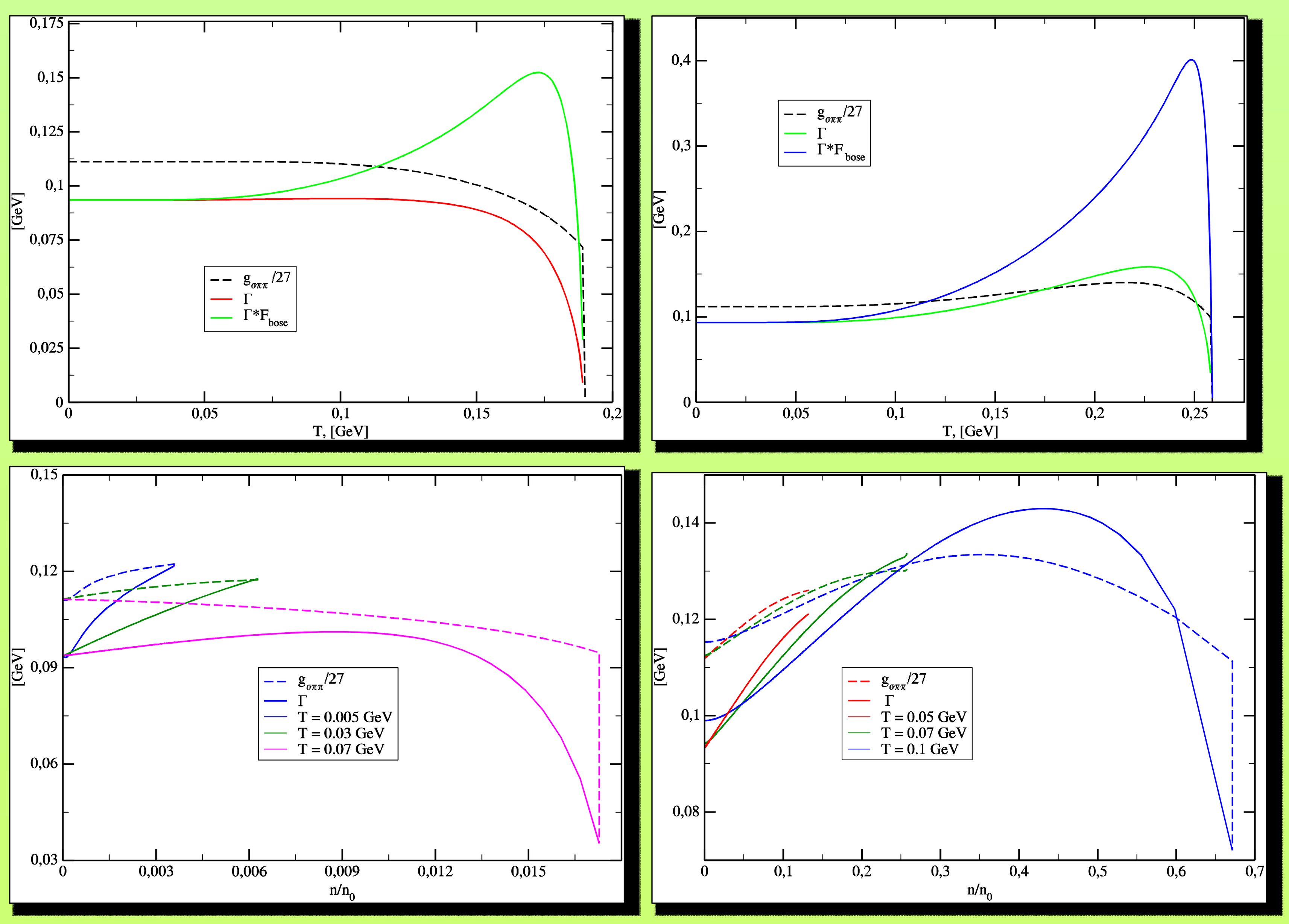


The mass spectra as a function of temperature (μ=0) and chemical potential (T = 5 MeV for NJL and T=9 MeV for PNJL)

The decay width $\Gamma(\sigma\to\pi\pi)$ is defined as[4]:

$$\Gamma(\sigma\to\pi\pi) = \frac{3}{32\pi}\frac{g_{\sigma\pi\pi}^2}{M_\sigma}\sqrt{1 - \frac{4M_\pi^2}{M_\sigma^2}}$$

NJL PNJL



The decay width $\Gamma(\sigma\to\pi\pi)$ and the decay constant $g_{\sigma\pi\pi}$ as a function of temperature and chemical potential

Conclusion

The PNJL model includes features of both deconfinement and chiral symmetry restoration. The model parameters are fixed by the pion decay constant, quark condensate and current quark mass. The PNJL model reproduces the temperature dependence of meson and quark masses and their widths more realistically than the NJL one. The obtained results may serve as a basis for studying a phase structure of QCD matter.

References

[1] H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi, C. Ratti , Phys.Rev.D75:065004, 2007.
[2] J. Hüfner, S.P. Klevansky, E.Quack, P. Zhuang, Phys.Lett. B337 (1994) 30-36.
[3] M. Buballa, Phys.Rept. 407 (2005) 205-376.
[4] P. Zhuang, Z. Yang, Chin.Phys.Lett. 18 (2001) 344-346.